



Classification of preferential ballot voting methods

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Abstract

We examine 17 preferential ballot voting methods using three criteria: (1) mathematical properties, (2) game-theoretic considerations, and (3) practical real-world outcomes. The challenge of comparing and contrasting the real-world outcomes of different voting methods is the sheer number of possible elections that can exist. We combat this challenge by introducing a new way of visualizing outcomes in what we dub a “DNA sequence” for each voting method.

Keywords Preferential ballot voting methods · mathematical properties · game theory

JEL Classification D720

1 Introduction

For a given number of candidates and political factions, how often will voting methods agree? In this article we address this question *visually*, by graphically representing tens of thousands of outcomes via “DNA sequences” for voting methods:



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The above figure is composed of five images, left to right, comparing 16 to 17 preferential ballot voting methods in each image. Note how these voting methods are roughly split into 3 groups, based on their appearance. We call these groups the “Global Ordering Methods,” the “Instant Runoff Methods,” and the “Points-Based Methods.” We describe how these DNA sequence images are computed in Sect. 3, preceded by brief descriptions of the 17 voting methods we have chosen to study in Sect. 2. We then briefly discuss the mathematical properties and game theories of preferential ballot voting methods in Sect. 4, which is very important for understanding which methods are ideal for various applications. Finally, in Sect. 5 we provide a list of the other fascinating contributions to this volume which, when taken together, reveal the rich diversity of perspectives that exist when trying to choose the ideal preferential ballot voting method for a given application.

2 List of voting methods

In the descriptions below, for the methods where candidates are successively eliminated, revotes among the remaining candidates are implied, as if the eliminated candidates had never existed.

Group I: Global Ordering Methods (outcomes similar to Ranked Pairs and Schulze)

1. *Ranked Pairs*, as defined in Nicolaus (1987).
2. *Instant Runoff Schulze* (“*IR-Sch.*”): The Schulze loser (winner according to Schulze after every ballot is inverted) among the remaining candidates is eliminated until only one candidate remains.
3. *Instant Runoff Greatest Victory* (“*IR-GV*”): The candidate with the smallest greatest pairwise victory among the remaining candidates is eliminated until only one candidate remains.
4. *Schulze* (“*Sch.*”), as defined in Schulze (2003).
5. *Smith Worst Defeat* (“*Sm.-WD*”): Worst Defeat, but from within the Smith set.
6. *Worst Defeat* (“*WD*” or “*minimax*”): The candidate with the least worst, worst defeat among pairwise matchups wins.

Group II: Instant Runoff Methods (outcomes similar to Smith Instant Runoff Voting)

7. *Instant Runoff Bottom-Two-Runoff* (“*IR-BTR*”): The loser of the head-to-head matchup between the two candidates with the fewest first place votes among the remaining candidates is eliminated until only one candidate remains.
8. *Instant Runoff Borda* (“*IR-Borda*”): The Borda Count loser (worst average ranking) among the remaining candidates is eliminated until only one candidate remains.
9. *Instant Runoff Worst Defeat* (“*IR-WD*”): The candidate with the worst pairwise defeat among the remaining candidates is eliminated until only one candidate remains.

10. *Smith Instant Runoff Voting* (“*Sm.-IRV*”): Instant Runoff Voting, but only from within the Smith set of the original candidates.
11. *Instant Runoff Smith* (“*IR-Sm.*,” also known as *Tideman’s Alternative*): Among the remaining candidates, alternate reducing to the Smith Set and eliminating the candidate with the fewest first place votes until only one candidate remains.
12. *Instant Runoff Condorcet* (“*IR-Cond.*”): Among the remaining candidates, alternate reducing to the Condorcet candidate (if one exists) and eliminating the candidate with the fewest first place votes until only one candidate remains. See (Green-Armytage, 2016).
13. *Instant Runoff Voting* (“*IRV*”): The candidate with the fewest first place votes among the remaining candidates is eliminated until only one candidate remains.

Group III: Points-Based Methods (Widely varying outcomes based on scoring points)

14. *Vote-for-one* (“*V-1*”): The candidate with the most first place votes wins. Points given by each voter to candidates in order of ranking are $(1, 0, 0, 0, \dots)$.
15. *Borda count*: The candidate with the best average ranking wins. The points given by each voter to N candidates in order of ranking are $(-1, -2, -3, \dots)$ or, equivalently, $(1 - \frac{1}{N}, 1 - \frac{2}{N}, 1 - \frac{3}{N}, \dots)$ (points between 0 and 1 for comparison purposes).
16. *Vote-for-two* (“*V-2*”): The candidate with the most first and second place votes wins. Points given by each voter to candidates in order of ranking are $(1, 1, 0, 0, 0, \dots)$.
17. *Vote-for-three* (“*V-3*”): The candidate with the most first, second, and third place votes wins. Points given by each voter to candidates in order of ranking are $(1, 1, 1, 0, 0, 0, \dots)$.

Grouping preferential ballot vote counting methods by outcomes instead of procedures is essential. For example, based on procedure, one might be tempted to group Instant Runoff Ranked Pairs in the “Instant Runoff Methods” group. However, Instant Runoff Ranked Pairs always gives the same outcome as Ranked Pairs, in every possible election, and hence should be considered the same preferential ballot voting method as Ranked Pairs.

The value of the DNA sequences we study in the next section is that they reveal that many methods are actually quite similar in terms of outcomes, at least for quite a few example election scenarios. For example, in the tens of thousands of scenarios covered by our DNA sequences, Ranked Pairs and Instant Runoff Schulze never disagreed (though they can in more complicated election scenarios), Schulze and Smith Worst Defeat rarely disagreed, and most instant runoff style methods gave similar results. A notable exception is Instant Runoff Greatest Victory, which, while not even being Condorcet, is more similar to the Global Ordering Methods, at least in the examples studied here. The Points-Based Methods, on the other hand, are the most varied in terms of outcomes, compared to the other two groups.

Table 1 36 possible elections among 3 candidates and 3 political factions of 5, 4, 2 voters

1.	2.	3.	4.	5.	6.
$\underline{5}$ $\underline{4}$ $\underline{2}$ A A A B B B C C C	$\underline{5}$ $\underline{4}$ $\underline{2}$ A A A B B C C C B	$\underline{5}$ $\underline{4}$ $\underline{2}$ A A B B B A C C C	$\underline{5}$ $\underline{4}$ $\underline{2}$ A A C B B A C C B	$\underline{5}$ $\underline{4}$ $\underline{2}$ A A B B B C C C A	$\underline{5}$ $\underline{4}$ $\underline{2}$ A A C B B B C C A
7.	8.	9.	10.	11.	12.
$\underline{5}$ $\underline{4}$ $\underline{2}$ A A A B C B C B C	$\underline{5}$ $\underline{4}$ $\underline{2}$ A A A C B B B C C	$\underline{5}$ $\underline{4}$ $\underline{2}$ A A B B C A C B C	$\underline{5}$ $\underline{4}$ $\underline{2}$ A A B C B A B C C	$\underline{5}$ $\underline{4}$ $\underline{2}$ A A B B C C C B A	$\underline{5}$ $\underline{4}$ $\underline{2}$ A A B C B C B C A
19.	20.	21.	22.	23.	24.
$\underline{5}$ $\underline{4}$ $\underline{2}$ A C A B A B C B C	$\underline{5}$ $\underline{4}$ $\underline{2}$ A B A C A B B C C	$\underline{5}$ $\underline{4}$ $\underline{2}$ A C B B A A C B C	$\underline{5}$ $\underline{4}$ $\underline{2}$ B A A C B B A C C	$\underline{5}$ $\underline{4}$ $\underline{2}$ A C B B A C C B A	$\underline{5}$ $\underline{4}$ $\underline{2}$ B A A C B C A C B
25.	26.	27.	28.	29.	30.
$\underline{5}$ $\underline{4}$ $\underline{2}$ A B A B C B C A C	$\underline{5}$ $\underline{4}$ $\underline{2}$ A B A B C C C A B	$\underline{5}$ $\underline{4}$ $\underline{2}$ B A A A C B C B C	$\underline{5}$ $\underline{4}$ $\underline{2}$ A B C B C A C A B	$\underline{5}$ $\underline{4}$ $\underline{2}$ C A A A B B B C C	$\underline{5}$ $\underline{4}$ $\underline{2}$ C A B A B A B C C
31.	32.	33.	34.	35.	36.
$\underline{5}$ $\underline{4}$ $\underline{2}$ A C A B B B C A C	$\underline{5}$ $\underline{4}$ $\underline{2}$ A B A C C B B A C	$\underline{5}$ $\underline{4}$ $\underline{2}$ B C A A A B C B C	$\underline{5}$ $\underline{4}$ $\underline{2}$ B A A C C B A C C	$\underline{5}$ $\underline{4}$ $\underline{2}$ C B A A A B B C C	$\underline{5}$ $\underline{4}$ $\underline{2}$ C A A B B B A C C

5, 4, 2 are underlined only for visual clarity

3 DNA sequences

All 36 possible elections among 3 candidates and 3 political factions with relative sizes 5, 4, and 2 are listed in Table 1. Note that we have renamed the candidates so that the Ranked Pairs ordering is A, B, C in every case. After numbering these $36 = (3!)^2$ election scenarios 1 through 36, we then compare preferential ballot voting methods by examining the winners of each of these 36 elections by each method, as shown in Table 2.

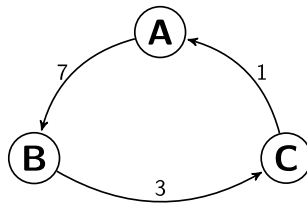
In Table 2, we have excluded Vote-for-Three since for three candidates it gives no information. Also, in several instances there were ties whose procedure for resolving we forego here. The complete agreement among RP, IR-Sch, IR-GV, Sch., Sm.-WD, and WD in Table 2 is not surprising, as these six methods always agree with 3 or fewer candidates.

Table 2 allows a person to pick their favorite voting method for a given application. For example, if one has narrowed the field to the top 12 methods listed above, then it all comes down to scenario 23, depicted by the graphic below:

Table 2 Winners of the elections in Table 1 via the first 16 voting methods in sect 2

Election#	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35		
Condorcet?	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✓	✓	✓	✓	✗	✓	✓	✓	✓	✓	✓	✓	✓	
RP	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	
IR-Sch.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	
IR-GV	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	
Sch.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	
Sm-WD	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	
WD	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	
IR-BTR	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	
IR-Borda	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	C	A	A	A	A	A	A	A	A	A	A	A	A	A	
IR-WD	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	C	A	A	A	A	A	A	A	A	A	A	A	A	A	A	
Sm-IRV	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	C	A	A	A	A	A	A	A	A	A	A	A	A	A	A	
IR-Sm.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	C	A	A	A	A	A	A	A	A	A	A	A	A	A	A	
IR-Cond	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	C	A	A	A	A	A	A	A	A	A	A	A	A	A	A	
IRV	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	C	A	A	A	A	A	A	A	A	A	A	A	A	A	B	
V-1	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	B	A	A	A	A	A	A	A	A	A	A	A	A	C	A	B	A	C		
Borda	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	B	A	B	A	B	B	A	A	B	A	A	A	A	A	A	A	
V-2	B	A	B	A	B	B	A	A	A	A	A	B	A	B	A	B	A	A	A	A	A	B	A	B	B	B	A	B	A	B	A	A	B	C	A	B	A

The deviations from A are in bold so as to be easy for the reader's eye to identify



where the arrow from A to B indicates that A defeated B head-to-head by a margin of 7, etc. Since we have a cycle of head-to-head preferences, this scenario is controversial. Arguments for each of the candidates include:

1. The argument for A is that A has the largest pairwise victory (7), the smallest pairwise defeat (1), the most first-place votes (5), and the fewest last-place votes (2).
2. The argument for C is that C beats A head-to-head, after B is eliminated for having the fewest first place votes, greatest pairwise defeat, and worst average rank.
3. None of our methods chose B as the winner in scenario 23, but if one could cook up a reason to eliminate A first, then B beats C in the runoff.

We leave it to the reader to determine who should win the election in scenario 23.

More generally, it is useful to turn Table 2 into graphical form:

Now suppose we consider 4 or 5 candidates, but still with 3 political factions with relative sizes of 5, 4, and 2, as before. The number of elections jumps to $(4!)^2 = 576$ and $(5!)^2 = 14,400$, respectively, so viewing the results graphically,

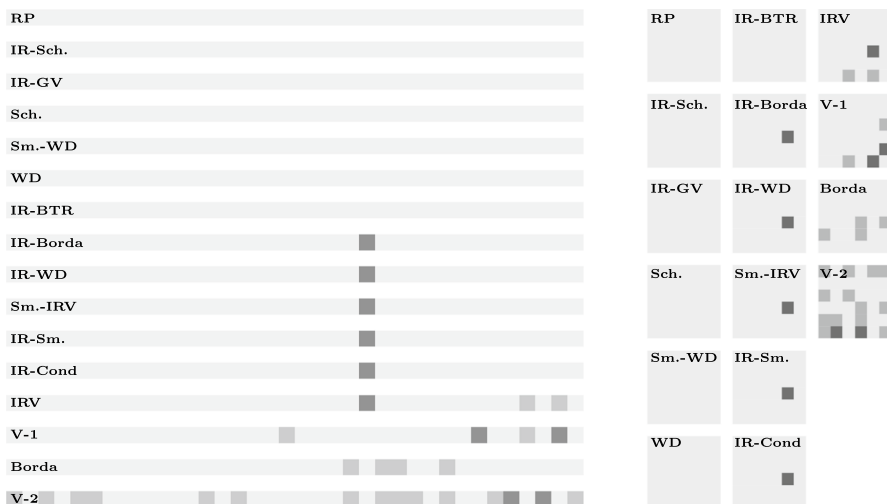


Fig. 1 The DNA sequences A,B,C are converted to shades of grey, with A the lightest, in a 1×36 arrangement on the left and a 6×6 arrangement on the right

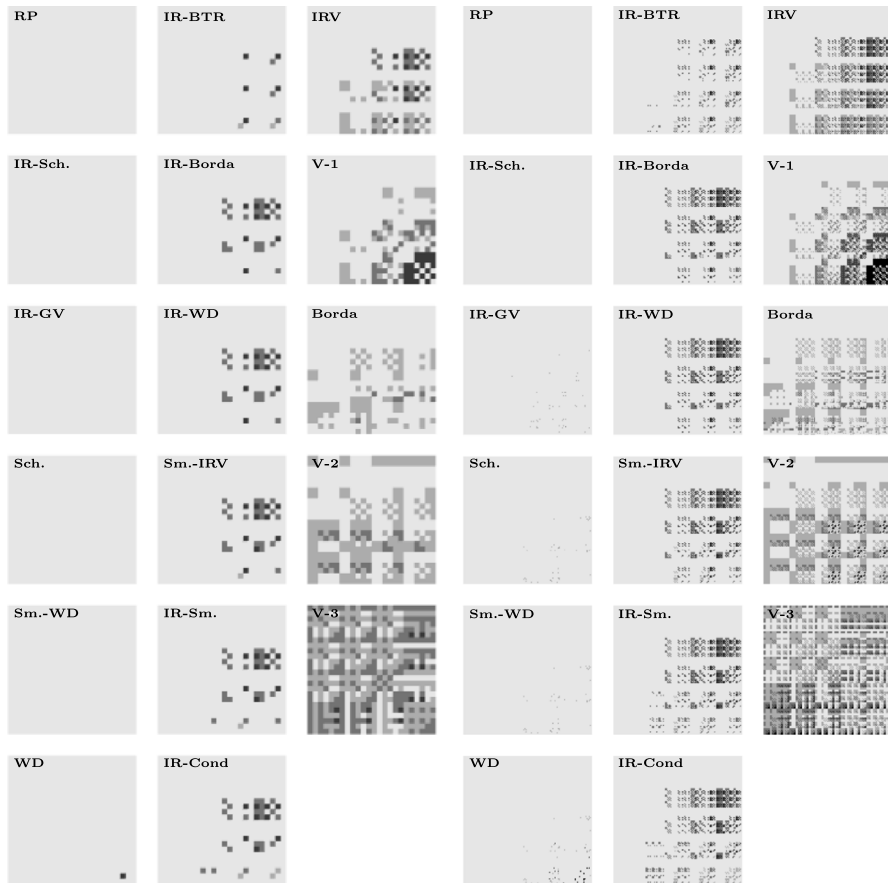


Fig. 2 DNA sequences for 4 candidates on the left (24×24) and 5 candidates on the right (120×120), with 3 political factions with relative sizes of 5, 4, and 2. The rows and columns of each square correspond to the permutations of the candidates in the second and third groups, respectively

as in Fig 2, is the only reasonable option. Because of computation time, we did not include the DNA sequences for 6 candidates and 3 political factions representing $(6!)^{3-1} = 518,400$ elections, nor did we include the DNA sequences for 5 candidates and 4 political factions representing $(5!)^{4-1} = 1,728,000$ elections. Figure 3, however, contains the DNA sequences for all 17 of our preferential ballot voting methods for 4 candidates and 4 political factions representing $(4!)^{4-1} = 13,824$ elections. Collectively, Figs 1, 2 and 3 reveal the similarities and differences of our 17 preferential ballot voting methods, at least in terms of outcomes for the scenarios considered.

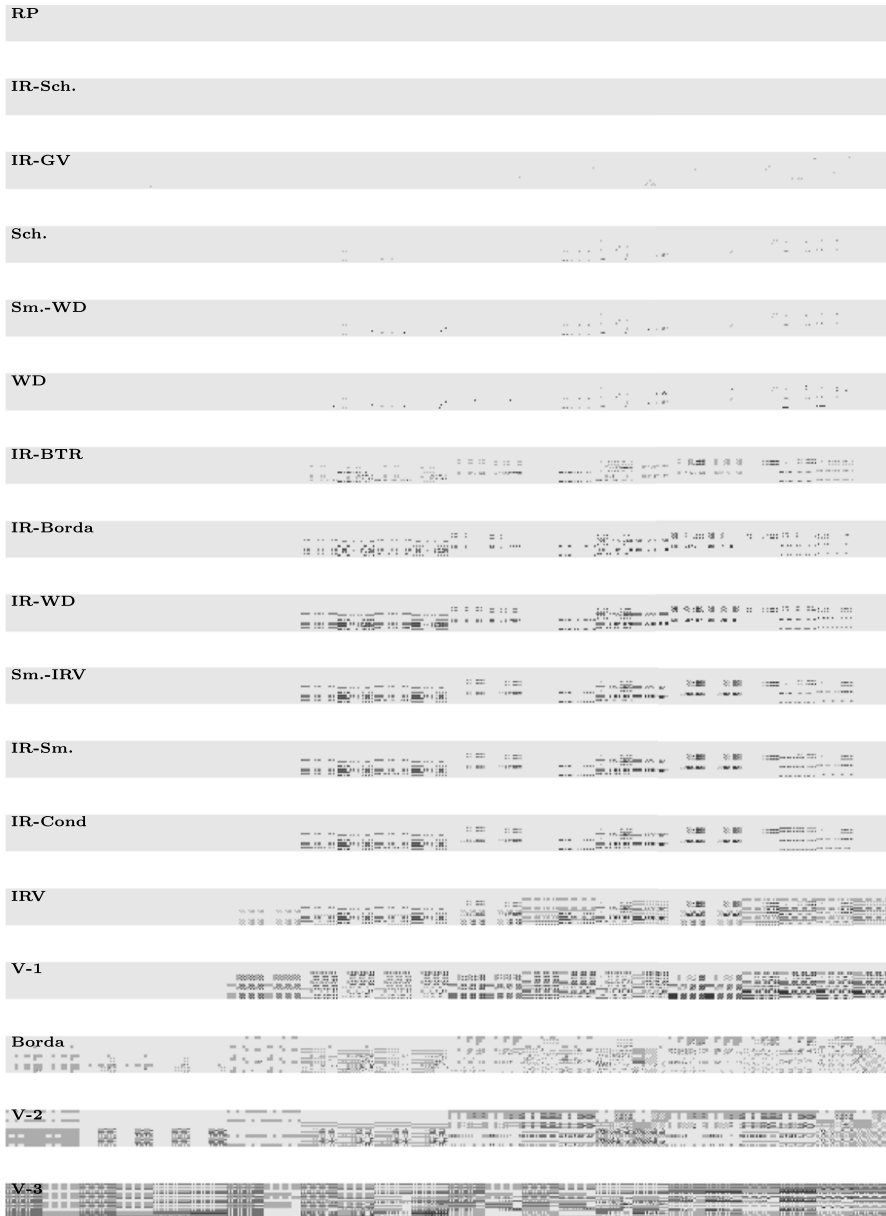


Fig. 3 DNA sequences for 4 candidates and 4 political factions with relative sizes of 8, 7, 6, and 4. Each DNA image is 24 squares left to right, each 24×24 . The square number corresponds to the permutation of the candidates in the second group, and the rows and columns of each square correspond to the permutations of the candidates in the third and fourth groups, respectively. In total, each image corresponds to $(4!)^3 = 13,824$ elections

4 Mathematical properties and game theory

When choosing an ideal voting method for a given application, it is also important to consider the mathematical properties and game-theoretic implications of each method.

Among the 17 preferential ballot voting methods in Sec. 2, Instant Runoff, Greatest Victory, Borda, Vote-for-Two, and Vote-for-Three don't satisfy the Majority Criterion, meaning that even if a candidate is the first choice of more than half the voters, that candidate is not guaranteed to win by these methods. If the voters in the election would find this upsetting, then this might be a problem and hence should be taken into account.

The "Nuclear War Scenario" (and its variations) are also a problem for the above methods, or potentially any other method that does not satisfy the Majority Criterion. We now demonstrate the issue with the Borda Count, as a prime example.

In this scenario, suppose there are 4 candidates in an election: Two candidates, A and B, are quite reasonable and popular, but two other candidates, X and Y, are disliked by everyone for the simple reason that they both want to start a nuclear war. X wants to drop 5 nuclear bombs, and Y wants to drop 10 nuclear bombs. The point of this scenario, however exaggerated it may appear, is that "nuclear war" stands for something none of the voters want. Hence, a voting method that leads to nuclear war, even though none of the voters wanted this, would be a bad voting method, to say the least.

In the Nuclear War Scenario, suppose 60% of the voters' preferences are A, B, X, Y and 40% of the voters' preferences are B, A, X, Y. Everyone ranks X and Y last, but X consistently above Y since Y is even worse than X.

The day before the election, it becomes clear to B that they are going to lose. Nevertheless, their game theory consultant tells them they can still win, if only they tell their voters to vote A last, so that 40% of the voters actually vote B, X, Y, A. Sure enough, on election day, B wins according to the Borda Count. B and their supporters are happy, and B's game theory consultant is applauded.

Not satisfying the Majority Criterion is what allows B to win, even though the majority of the voters clearly prefer A. Those who believe democracy is more important than game theory will not like the above outcome. But the true problem is much worse than this. Indeed, suppose that A had employed B's strategy as well. Then 60% of the voters would vote A, X, Y, B and 40% would vote B, X, Y, A. Now on election day, X wins according to the Borda Count! None of the voters wanted nuclear war, yet since dishonest voting is sometimes incentivized, and the Majority Criterion was not there to stop it, A does not win, in spite of receiving 60% of the first choice votes, and nuclear war results instead.

The Condorcet property is a further strengthening of the Majority Criterion. In elections with many candidates, it might be quite unlikely that any one candidate receives a majority of the first choice votes. Even so, there might still be a candidate, called the Condorcet candidate, who defeats every other candidate in a head-to-head matchup. A preferential ballot vote counting method that always selects the Condorcet candidate, when one exists, as the winner is called a Condorcet method.

The Condorcet property is additional insurance protecting voters from the Nuclear War Scenario. Suppose we still have a 60-40 split in the voters who vote A, B, X, Y and B, A, X, Y, respectively, but with a twist: Write-in votes are allowed, and every voter votes for themselves as their first choice, with only a few exceptions: Out of modesty, neither A nor B vote in the election (and hence receive no first choice votes) and somehow, X manages to trick 3 voters to vote for X as their first choice, and Y manages to trick 2 voters to vote for Y as their first choice. Suppose the winner is determined by either Vote-for-One (Plurality) or Instant Runoff Voting, which both satisfy the Majority Criterion. Then X wins, and we have nuclear war once again, in spite of the fact that A was a Condorcet candidate. Any Condorcet method would have chosen A as the winner, thereby avoiding nuclear war, something that only a few voters actually voted for.

Another game theoretic scenario to consider is the Media Manipulation Scenario. Suppose 99 voters rank the candidates A, B, C, D, but that one other voter, who owns the local media, ranks the candidates D, C, B, A. If the single voter who controls the local media can convince everyone that the true contest is between C and D, and if Approval Voting is being used to determine the winner, then the 99 voters will all vote for A, B, C, since they think that to not vote for either C or D is to throw their vote away. If the media candidate then votes for D and C, then according to Approval Voting, C will win the election, in spite of A being preferred by 99% of the voters.

More generally, Approval Voting, where voters may vote for any number of candidates, should be thought of as something similar to the Vote-for-Two (or Three, etc.) methods, at least in some regards. In practice, one study (Brams & Nagel, 1991) found that voters voted, on average, for 1.52 candidates in an election with 4 candidates. As far as the instructions that voters should vote for all of the candidates that they “approve” of, what if the voter approves of no one or everyone? Will they really turn in a blank ballot, or a ballot where they vote for everyone? If not, then this forces voters to think in a game-theoretic manner.

In addition to the Majority Criterion, three other well known properties are Clone Invariance, Monotonicity, and Last Place Loser Independence.

Clone Invariance means that a candidate is neither rewarded nor penalized if an identical candidate to themselves is added to the election. Famously, Vote-for-One (Plurality) is not Clone Invariant. Duverger’s Law (Duverger, 1959) explains why Vote-for-One elections typically have precisely two dominant political parties so as to not “split the vote.” Instant Runoff Voting (IRV) is often viewed as the “fix” for this as it is Clone Invariant. However, there are many Clone Invariant methods from which to choose, chief among them being Ranked Pairs and the Schulze method.

IRV is not Monotone, however. This means a candidate can lose, amazing as it may sound, from getting too many votes! For example, suppose 8 vote ABC, 7 vote BCA, and 6 vote CAB. Then A wins according to IRV. Now suppose A does some last minute campaigning, and convinces two voters to change their votes from BCA to ABC. Now 10 vote ABC, 5 vote BCA, and 6 vote CAB, and C wins according to IRV!

Last Place Loser Independence is the final property we mention. Define the Last Place Loser to be the winner of the election, if every voter inverted their ranking of

the candidates. That is, the Last Place Loser is the candidate who wins the election for worst candidate. A preferential ballot voting method is “Last Place Loser Independent” if the winner never changes if the Last Place Loser drops out of the election. This property helps to decrease the bargaining power of candidates who are not popular but who might threaten to drop out of the election to change its outcome.

The only preferential ballot voting method we are aware of that is Condorcet, Clone Invariant, Monotone, and Last Place Loser Independent is Ranked Pairs. Generally speaking, as each property protects from game theoretic scenarios, more mathematical properties are arguably beneficial.

Ranked Pairs is also explicitly designed to give an ordering of the candidates, not just a winner. In fact, Ranked Pairs can be defined as selecting the ordering of the candidates which minimizes the preferences, raised to the power p , summed over the pairwise matchups that go against the given ordering, in the limit as p goes to infinity. Note that p must be sufficiently large for the resulting method to be Clone Invariant, so the above description for Ranked Pairs is well motivated. In our list of 17 methods, Ranked Pairs is the only method that explicitly optimizes an overall ordering. Notably, when using Ranked Pairs, the 1st place candidate always beats the 2nd place candidate, 2nd always beats 3rd, 3rd always beats 4th, etc., in head-to-head matchups. This is another very nice property, as a 2nd place candidate might object if they beat the 1st place candidate in a head-to-head matchup.

Concerning voting applications where, for example, the top five candidates need to be chosen, Ranked Pairs is also particularly appropriate. With most voting methods, the order of the winners can change when the last place candidate is removed. Hence, the “top five” are not uniquely defined: Do you mean the top five with all the candidates in the election, or the top five remaining if we remove candidates one at a time? With Ranked Pairs, the answers to these questions are always the same.

On the other hand, Smith Worst Defeat almost always gives the same answer as Ranked Pairs, and is arguably easier to understand than Ranked Pairs. But it does not give an overall ordering to the candidates liked Ranked Pairs. What do you do if the winner is disqualified somehow? These rules would need to be clearly explained in advance as there is more than one possible solution. For Ranked Pairs there is not only a winner but an ordering for all of the candidates, so it is clearer how all the candidates placed. For applications that need a clear 1st place, 2nd place, 3rd place, etc., Ranked Pairs is arguably most transparent.

The Schulze method also gives something close to an ordering, but can have generic ties among the candidates, not counting the overall winner and loser. These generic ties are not canonically broken, which could cause controversy, if an overall ordering is important. Curiously, Instant Runoff Schulze is much closer to Ranked Pairs than to Schulze, and is almost indistinguishable from Ranked Pairs, according to our DNA analysis.

In summary, in this paper we classified preferential ballot voting methods into three groups of methods: Points-Based Methods, Instant Runoff Methods, and Global Ordering Methods. The Points-Based Methods like Borda and Vote-for-Two are generally vulnerable to the Nuclear War Scenario described above and hence should not be used to elect the leaders of any country with a nuclear arsenal (or for

applications where dishonest voting is a concern). In this sense, Borda, Vote-for-Two, and Vote-for-Three are worse than Vote-for-One (Plurality) which satisfies the Majority Criterion and hence is not vulnerable to the first Nuclear War Scenario. However, to be immune to the second Nuclear War Scenario, one would need to restrict to Condorcet methods. Many of the Instant Runoff Methods are Condorcet, but these methods tend not to be Monotone, so that a candidate can sometimes lose by getting too many votes. By contrast, many of the Global Ordering Methods are both Condorcet and Monotone. The fact that Ranked Pairs provides not only a winner but an ordering of all of the candidates that does not change when the top or bottom candidates are removed (known as the “local IIA” property) makes Ranked Pairs a great choice for many applications.

5 What is in this volume?

We conclude our article by briefly summarizing the wonderful contributions to this volume, written by experts spanning several disciplines:

2. C. Song performs an exhaustive analysis of data from Politbarometer surveys and from actual elections (both German and American), in order to measure the differences in outcomes across voting methods. Interestingly, Song finds that the two voting methods most similar to each other in outcomes were *Worst Defeat* and *STAR voting*.
3. How does a voting method incentivize the winner to behave *after* the election? Motivated by this question, and by the effect that party primaries have on election outcomes, K. Gehl argues that *Final Five Voting*, which is an open top-five primary followed by IRV, is the best equipped to deliver the most democratic outcomes.
4. W. Smith makes a case for *score voting*, based on its performance in simulations (calculating Bayesian regret), on biological factors (certain species of ants and bees use score voting), and on its simplicity and ease of use.
5. Staying within score voting, S. Wolk, J. Quinn, & M. Ogren advocate for *STAR voting* (as well as for *Smith-Worst Defeat*), making a statistical argument that it does the best job of maximizing “Later no Harm” while minimizing “Favorite Betrayal.”¹
6. R. Laraki also advocates for score voting, but with adjectives in place of numbers (e.g., “Excellent,” “Very good,” ..., “Inadequate”), and for assigning to each candidate a “majority grade,” namely, the highest adjective that is approved or bettered by a majority. Termed *majority judgment* voting, Laraki argues that

¹ “Later no Harm” describes the case when endorsing candidates other than one’s favorite will not hurt one’s favorite candidate later in the election; “Favorite Betrayal” is the practice of voting for a candidate other than one’s favorite because the latter is perceived to not have a realistic chance of winning.

aside from the usual benefits of score voting, this modification is better equipped to avoid the “Domination Paradox.”²

7. Yet another case for score voting is made by Hamlin & Hua: Centering their argument on the administration, transparency, and implementation of elections, as well as on voter satisfaction, they advocate for *approval voting* as the most effective.
8. Preferring positional voting methods to those that rely on the “silos” of pairwise voting, D. Saari advocates for the *Borda count*, arguing that it is the most natural extension of a two-candidate majority vote, and that among the positional voting methods, it is the one least susceptible to game theory.
9. Encouraged by the increasing (worldwide) adoption of *IRV*, as well as the fact that it satisfies clone invariance, R. Richie, J. Seitz-Brown, & L. Kaufman find compelling *IRV*’s promotion of majority rule and its rewarding of sincere voting.
10. Arguing for the importance of using spatial models as in Nicolaus and Plassmann (2010), R. Robinette provides a careful examination of the extent to which strategic positioning/voting can alter the outcome in *IRV* and *Worst Defeat* elections. Among other things, Robinette advises against moving survey or ballot data “across” different voting methods.
11. Taking a more cautious view of *IRV*, R. Bristow-Johnson urges us not to forget the 2009 mayoral election of Burlington, Vermont, wherein *IRV* did not elect the Condorcet winner. Motivated by this, Bristow-Johnson advocates for *BTR-IRV*, which is Condorcet.
12. Via exhaustive simulations, F. Durand argues that the *IRV* family exhibits the smallest rate of manipulation by coalitions of voters, where “manipulation” is defined and considered in several different ways.
13. Staying within the global pairwise family, and privileging simplicity in a voting method, R. Darlington advocates for a version of Smith-WD called *Mini-max -TD*, wherein equal or absent ranks are allowed. Based on extensive simulations, Darlington argues that it is as effective as either Ranked Pairs or Schulze, but simpler than both.
14. Guided by the rationale that a larger margin of victory should outweigh a smaller one, C. Munger, Jr. advocates for *Ranked Pairs*, finding most compelling its combination of clone invariance and the fact that eliminating candidates would not alter the remaining rank order (via the last place loser independence property shown in Table 1).
15. W. Holliday & E. Pacuit propose a new Condorcet-compatible voting method, called *stable voting*, and show, among other things, that, unlike plurality, *IRV*, and the Schulze method, its tendency to produce ties *decreases* as the number of candidates increases; furthermore, stable voting, unlike Worst Defeat, has the property that its winner must come from the Smith set.
16. Arguing for the need to be able to arrive at a consensus *during* an election, J. Green-Armytage proposes a Condorcet method that allows voters to vote again from within the Smith set until a Condorcet winner emerges. During

² The “Domination Paradox” is the case when the majority-preferred candidate is nevertheless less passionately preferred by that majority than is a minority candidate by their supporters.

this process, candidates are allowed to withdraw and a procedure is given for resolving “endless” cycles. This method is then compared with other well known voting methods.

17. Finally, N. Tideman’s concluding article, in exploring the game theory of all of these voting methods, provides both a deep and broad view of the voting landscape.

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Declarations

Conflict of interest The authors declare no competing interests

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